Commuting Linear Maps on a Kind of Heisenberg - Virasoro Lie Algebra

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Abstract: Using the properties of commuting linear maps, we calculate the commuting linear maps of the Heisenberg - Virasoro Lie algebra of rank two, which is helpful to determine the centroid of the Lie algebra.

Keywords: Heisenberg - Virasoro Lie algebra of rank two; commuting linear map; center

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0 Introduction

The structures and representations of the Heisenberg - Virasoro Lie algebra and its generalizations have been widely studied, such as [1]~[7]. The Heisenberg - Virasoro Lie algebra of rank one is the Lie algebra of differential operators of order at most one [1], which is generalized to the case of rank two in [8]. Some structure properties of the Heisenberg - Virasoro Lie algebra of rank two have been calculated in [8], including central extensions, derivations and the automorphism group. In [9], $\mathbb{Z}$-graded Harish - Chandra modules of the Heisenberg - Virasoro Lie algebra of rank two was classified. Now there are still many questions unknown about this Lie algebra.

Linear maps on Lie algebras are important part of the structure theory of Lie algebras, such as derivation, endomorphism and automorphism and so on. Commuting linear maps on Lie algebra is a special kind of linear maps, and it is an indispensable part of the structure theory of Lie algebras. In this paper, we mainly study the commuting linear maps on the Heisenberg - Virasoro Lie algebra of rank two.

Throughout this paper, we denote by $\mathbb{Z}$, $\mathbb{Z}^*$, $\mathbb{Q}$, $\mathbb{Q}^*$ and the $\mathbb{C}$ set of integers, non - zero integers, rational numbers, non - zero rational numbers and complex numbers, respectively. All the vector spaces are assumed over the complex field.

1 Preliminary

Let $G=\{\alpha=(\alpha_1,\alpha_2)\mid\alpha_1,\alpha_2\in\mathbb{Z},\alpha\neq(0,0)\}$ and $\mathbb{Q}^*=\mathbb{Q}\backslash\{0,-1\}$. Set

$$\eta=(1,0),\varepsilon=(0,1).$$

For $\alpha=(\alpha_1,\alpha_2),\beta=(\beta_1,\beta_2)\in G$, denote by

$$\det(\beta,\alpha)=\begin{vmatrix}\beta_1 & \beta_2\\ \alpha_1 & \alpha_2\end{vmatrix}=\beta_1\alpha_2-\beta_2\alpha_1.$$

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Obviously, $\det(\beta,\alpha)=0$ if and only if the set $\{\alpha,\beta\}$ is linear dependent, and $\det(\beta,\alpha)\neq 0$ if and only if the set $\{\alpha,\beta\}$ is linear independent.

For any $\alpha\in G$, set

$$\mathscr{L}(\alpha)=\{q\alpha\in G \mid q\in\mathbb{Q}^*\}.$$

Then, for $\alpha,\beta\in G$, we have $\det(\beta,\alpha)=0$ if and only if $\beta\in\mathscr{L}(\alpha)$, and $\det(\beta,\alpha)\neq 0$ if and only if $\beta\notin\mathscr{L}(\alpha)$.

Definition 1[8-9] The Heisenberg - Virasoro Lie algebra of rank two, denoted by $\mathfrak{g}$, has a basis

$$\{t^\alpha, E_\alpha, K_i \mid \alpha\in G, i=1,2,3,4\}$$

with the following brackets:

$$[t^\alpha,t^\beta]=0,\quad [K_i,\mathfrak{g}]=0,\quad i=1,2,3,4,$$
$$[t^\alpha,E_\beta]=\det(\beta,\alpha)t^{\alpha+\beta}+\delta_{\alpha+\beta,0}(\alpha_1K_1+\alpha_2K_2),$$
$$[E_\alpha,E_\beta]=\det(\beta,\alpha)E_{\alpha+\beta}+\delta_{\alpha+\beta,0}(\alpha_1K_3+\alpha_2K_4)$$

for all $\alpha,\beta\in G$.

Obviously, the center of the Heisenberg - Virasoro Lie algebra of rank two $C(\mathfrak{g})=\mathbb{C}K_1\oplus\mathbb{C}K_2\oplus\mathbb{C}K_3\oplus\mathbb{C}K_4$. According to Definition 1, when the set $\{\alpha,\beta\}$ is linear dependent and $\beta\neq-\alpha$, we have

$$[t^\alpha,t^\beta]=[t^\alpha,E_\beta]=[E_\alpha,E_\beta]=0.$$

2 Commuting linear maps on the Lie algebra $\mathfrak{g}$

In this section, we determine commuting linear maps on the Heisenberg - Virasoro Lie algebra of rank two $\mathfrak{g}$.

Definition 2[10] Let L be a Lie algebra over the field $\mathbb{F}$. A linear map $f: L\to L$ is called a commuting linear map of the Lie algebra L if

$$[x,f(x)]=0$$

for all $x\in L$.

For any commuting linear map f of a Lie algebra L, it is easy to see that

$$f(K_i)\in C(\mathfrak{g}),\quad i=1,2,3,4;$$
$$[f(x),y]=[x,f(y)],\quad \forall\, x,y\in L.$$

Theorem 1 For any commuting linear map f on the Heisenberg - Virasoro Lie algebra of rank two $\mathfrak{g}$, there exist some $a,b,x_i^\alpha,y_i^\alpha,z_i(j)\in\mathbb{C}(i,j=1,2,3,4)$ such that

$$f(t^\alpha)=at^\alpha+\sum_{i=1}^4 x_i^\alpha K_i,\ f(E_\alpha)=bt^\alpha+aE_\alpha+\sum_{i=1}^4 y_i^\alpha K_i,\ f(K_i)=\sum_{j=1}^4 z_i(j)K_j$$

for all $\alpha\in G$.

Proof Let f be any commuting linear map on $\mathfrak{g}$. For any $\alpha\in G$, assume

$$f(t^\alpha)=\sum\lambda_{\gamma_\alpha}t^{\gamma_\alpha}+\sum\mu_{\theta_\alpha}E_{\theta_\alpha}+\sum_{i=1}^4 x_i^\alpha K_i,$$
$$f(E_\alpha)=\sum\rho_{\xi_\alpha}t^{\xi_\alpha}+\sum\sigma_{\zeta_\alpha}E_{\zeta_\alpha}+\sum_{i=1}^4 y_i^\alpha K_i,$$
$$f(K_i)=\sum_{j=1}^4 z_i(j)K_j,$$

where $\gamma_\alpha,\theta_\alpha,\xi_\alpha,\zeta_\alpha\in G$ and $\lambda_{\gamma_\alpha},\mu_{\theta_\alpha},\rho_{\xi_\alpha},\sigma_{\zeta_\alpha},x_i^\alpha,y_i^\alpha,z_i(j)\in\mathbb{C}$.

Since $[t^\alpha,f(t^\alpha)]=0$, we have

$$0=\sum\lambda_{\gamma_\alpha}[t^\alpha,t^{\gamma_\alpha}]+\sum\mu_{\theta_\alpha}[t^\alpha,E_{\theta_\alpha}]=$$

$$\sum \mu_{\theta_\alpha}[\det(\theta_\alpha,\alpha)t^{\alpha+\theta_\alpha}+\delta_{\alpha+\theta_\alpha,0}(\alpha_1K_1+\alpha_2K_2)]=$$
$$\sum \mu_{\theta_\alpha}\det(\theta_\alpha,\alpha)t^{\alpha+\theta_\alpha}+\mu_{-\alpha}(\alpha_1K_1+\alpha_2K_2),$$

which forces

$$\det(\theta_\alpha,\alpha)\mu_{\theta_\alpha}=0, \tag{1}$$
$$\alpha_1\mu_{-\alpha}=0,\quad \alpha_2\mu_{-\alpha}=0. \tag{2}$$

Since $\alpha=(\alpha_1,\alpha_2)\neq(0,0)$, by(2), we have

$$\mu_{-\alpha}=0.$$

Using(1), it is easy to see $\mu_{\theta_\alpha}=0$ if $\det(\theta_\alpha,\alpha)\neq 0$, that is, $\mu_{\theta_\alpha}=0$ if $\theta_\alpha\notin\mathcal{L}(\alpha)$.
Hence, we can write

$$f(t^\alpha)=\sum\lambda_{\gamma_\alpha}t^{\gamma_\alpha}+\sum_{q\in\mathbb{Q}^*}\mu_{q\alpha}E_{q\alpha}+\sum_{i=1}^4 x_i^\alpha K_i. \tag{3}$$

Since $[E_\alpha,f(E_\alpha)]=0$, we have

$$0=\sum\rho_{\xi_\alpha}[E_\alpha,t^{\xi_\alpha}]+\sum\sigma_{\zeta_\alpha}[E_\alpha,E_{\zeta_\alpha}]=$$
$$-\sum\rho_{\xi_\alpha}[\det(\alpha,\xi_\alpha)t^{\alpha+\xi_\alpha}-\delta_{\alpha+\xi_\alpha,0}(\alpha_1K_1+\alpha_2K_2)]+$$
$$\sum\sigma_{\zeta_\alpha}[\det(\zeta_\alpha,\alpha)E_{\alpha+\zeta_\alpha}+\delta_{\alpha+\zeta_\alpha,0}(\alpha_1K_3+\alpha_2K_4)]=$$
$$-\sum\rho_{\xi_\alpha}\det(\alpha,\xi_\alpha)t^{\alpha+\xi_\alpha}+\rho_{-\alpha}(\alpha_1K_1+\alpha_2K_2)+$$
$$\sum\sigma_{\zeta_\alpha}\det(\zeta_\alpha,\alpha)E_{\alpha+\zeta_\alpha}+\sigma_{-\alpha}(\alpha_1K_3+\alpha_2K_4).$$

Then we have

$$\det(\alpha,\xi_\alpha)\rho_{\xi_\alpha}=0, \tag{4}$$
$$\alpha_1\rho_{-\alpha}=0,\quad \alpha_2\rho_{-\alpha}=0, \tag{5}$$
$$\det(\zeta_\alpha,\alpha)\sigma_{\zeta_\alpha}=0, \tag{6}$$
$$\alpha_1\sigma_{-\alpha}=0,\ \alpha_2\sigma_{-\alpha}=0. \tag{7}$$

Since $\alpha=(\alpha_1,\alpha_2)\neq(0,0)$, by (5) and (7), we have

$$\rho_{-\alpha}=0,\quad \sigma_{-\alpha}=0.$$

By (4), we have $\rho_{\xi_\alpha}=0$ if $\xi_\alpha\notin\mathcal{L}(\alpha)$. By (6), we have $\sigma_{\zeta_\alpha}=0$ if $\zeta_\alpha\notin\mathcal{L}(\alpha)$. Then we can write

$$f(E_\alpha)=\sum_{s\in\mathbb{Q}^*}\rho_{s\alpha}t^{s\alpha}+\sum_{k\in\mathbb{Q}^*}\sigma_{k\alpha}E_{k\alpha}+\sum_{i=1}^4 y_i^\alpha K_i. \tag{8}$$

Using $[f(t^\alpha),E_\alpha]=[t^\alpha,f(E_\alpha)]$, by (3) and (8), since

$$[f(t^\alpha),E_\alpha]=\sum\lambda_{\gamma_\alpha}[\det(\alpha,\gamma_\alpha)t^{\alpha+\gamma_\alpha}-\delta_{\alpha+\gamma_\alpha,0}(\alpha_1K_1+\alpha_2K_2)]+$$
$$\sum_{q\in\mathbb{Q}^*}\mu_{q\alpha}[\det(\alpha,q\alpha)E_{\alpha+q\alpha}+\delta_{\alpha+q\alpha,0}(q\alpha_1K_3+q\alpha_2K_4)]=$$
$$\sum\lambda_{\gamma_\alpha}\det(\alpha,\gamma_\alpha)t^{\alpha+\gamma_\alpha}-\lambda_{-\alpha}(\alpha_1K_1+\alpha_2K_2),$$
$$[t^\alpha,f(E_\alpha)]=\sum_{k\in\mathbb{Q}^*}\sigma_{k\alpha}[\det(k\alpha,\alpha)t^{\alpha+k\alpha}+\delta_{\alpha+k\alpha,0}(\alpha_1K_1+\alpha_2K_2)]=0,$$

we have

$$\sum\lambda_{\gamma_\alpha}\det(\alpha,\gamma_\alpha)t^{\alpha+\gamma_\alpha}-\lambda_{-\alpha}(\alpha_1K_1+\alpha_2K_2)=0.$$

Obviously, $\lambda_{-\alpha}=0$, and $\lambda_{\gamma_\alpha}=0$ if $\gamma_\alpha\notin\mathcal{L}(\alpha)$. Then (3) becomes

$$f(t^\alpha)=\sum_{p\in\mathbb{Q}^*}\lambda_{p\alpha}t^{p\alpha}+\sum_{q\in\mathbb{Q}^*}\mu_{q\alpha}E_{q\alpha}+\sum_{i=1}^4 x_i^\alpha K_i. \tag{9}$$

Using $[f(t^{\alpha}),t^{\beta}]=[t^{\alpha},f(t^{\beta})]$, by (9), since

$$[f(t^{\alpha}),t^{\beta}]=-\sum_{q\in\mathbb{Q}^*}\mu_{q\alpha}[\det(q\alpha,\beta)t^{\beta+q\alpha}+\delta_{\beta+q\alpha,0}(\beta_1K_1+\beta_2K_2)],$$

$$[t^{\alpha},f(t^{\beta})]=\sum_{q\in\mathbb{Q}^*}\mu_{q\beta}[\det(q\beta,\alpha)t^{\alpha+q\beta}+\delta_{\alpha+q\beta,0}(\alpha_1K_1+\alpha_2K_2)],$$

we get

$$-\sum_{q\in\mathbb{Q}^*}\mu_{q\alpha}[\det(q\alpha,\beta)t^{\beta+q\alpha}+\delta_{\beta+q\alpha,0}(\beta_1K_1+\beta_2K_2)]=$$
$$\sum_{q\in\mathbb{Q}^*}\mu_{q\beta}[\det(q\beta,\alpha)t^{\alpha+q\beta}+\delta_{\alpha+q\beta,0}(\alpha_1K_1+\alpha_2K_2)]. \tag{10}$$

Take $\beta\notin\mathbb{Z}(\alpha)$ in (10), then the set$\{\alpha,\beta\}$ is linear independent. So $\det(q\alpha,\beta)\neq0,\beta+q\alpha\neq0,\det(q\beta,\alpha)\neq0$ and $\alpha+q\beta\neq0$ for any $q\in\mathbb{Q}^*$, which forces (10)becomes

$$-\sum_{q\in\mathbb{Q}^*}\mu_{q\alpha}\det(q\alpha,\beta)t^{\beta+q\alpha}=\sum_{q\in\mathbb{Q}^*}\mu_{q\beta}\det(q\beta,\alpha)t^{\alpha+q\beta}. \tag{11}$$

Then

$$\mu_{q\alpha}\det(q\alpha,\beta)t^{\beta+q\alpha}=0,\quad q\neq0,\pm1.$$

It is easy to see that

$$\mu_{q\alpha}=0,\quad q\neq0,\pm1.$$

Hence (9) becomes

$$f(t^{\alpha})=\sum_{p\in\mathbb{Q}^*}\lambda_{p\alpha}t^{p\alpha}+\mu_{\alpha}E_{\alpha}+\sum_{i=1}^{4}x_i^{\alpha}K_i. \tag{12}$$

It follows from (11) that

$$\det(\alpha,\beta)(\mu_{\beta}-\mu_{\alpha})t^{\beta+\alpha}=0.$$

If the set$\{\alpha,\beta\}$ is linear independent, then $\det(\alpha,\beta)\neq0$ and

$$\mu_{\beta}=\mu_{\alpha}.$$

Then $\mu_{\alpha}=\mu_{\varepsilon}$ for $\alpha\neq l\varepsilon$, and $\mu_{l\varepsilon}=\mu_{\eta}=\mu_{\varepsilon}$, where $l\in\mathbb{Z}^*$. So

$$\mu_{\alpha}=\mu_{\varepsilon},\quad\forall\alpha\in G.$$

Hence (12) becomes

$$f(t^{\alpha})=\sum_{p\in\mathbb{Q}^*}\lambda_{p\alpha}t^{p\alpha}+\mu_{\varepsilon}E_{\alpha}+\sum_{i=1}^{4}x_i^{\alpha}K_i. \tag{13}$$

Using $[f(t^{\alpha}),E_{\beta}]=[t^{\alpha},f(E_{\beta})]$, by (8) and (13), since

$$[f(t^{\alpha}),E_{\beta}]=[\sum_{p\in\mathbb{Q}^*}\lambda_{p\alpha}t^{p\alpha}+\mu_{\varepsilon}E_{\alpha}+\sum_{i=1}^{4}x_i^{\alpha}K_i,E_{\beta}]=$$
$$\sum_{p\in\mathbb{Q}^*}\lambda_{p\alpha}[\det(\beta,p\alpha)t^{p\alpha+\beta}+\delta_{p\alpha+\beta,0}(p\alpha_1K_1+p\alpha_2K_2)]+$$
$$\mu_{\varepsilon}[\det(\beta,\alpha)E_{\alpha+\beta}+\delta_{\alpha+\beta,0}(\alpha_1K_3+\alpha_2K_4)],$$

$$[t^{\alpha},f(E_{\beta})]=[t^{\alpha},\sum_{s\in\mathbb{Q}^*}\rho_{s\beta}t^{s\beta}+\sum_{k\in\mathbb{Q}^*}\sigma_{k\beta}E_{k\beta}+\sum_{i=1}^{4}y_i^{\beta}K_i]=$$
$$\sum_{k\in\mathbb{Q}^*}\sigma_{k\beta}[\det(k\beta,\alpha)t^{\alpha+k\beta}+\delta_{\alpha+k\beta,0}(\alpha_1K_1+\alpha_2K_2)],$$

we have

$$\sum_{p\in\mathbb{Q}^*}\lambda_{p\alpha}[\det(\beta,p\alpha)t^{p\alpha+\beta}+\delta_{p\alpha+\beta,0}(p\alpha_1K_1+p\alpha_2K_2)]+$$
$$\mu_{\varepsilon}[\det(\beta,\alpha)E_{\alpha+\beta}+\delta_{\alpha+\beta,0}(\alpha_1K_3+\alpha_2K_4)]=$$

$$\sum_{k\in\mathbb{Q}^*}\sigma_{k\beta}[\det(k\beta,\alpha)t^{\alpha+k\beta}+\delta_{\alpha+k\beta,0}(\alpha_1K_1+\alpha_2K_2)]. \tag{14}$$

Taking $\beta=-\alpha$ in (14), we get $\mu_\varepsilon(\alpha_1K_3+\alpha_2K_4)=0$. Hence

$$\mu_\varepsilon=0 \tag{15}$$

and (14) becomes

$$\sum_{p\in\mathbb{Q}^*}\lambda_{p\alpha}[\det(\beta,p\alpha)t^{p\alpha+\beta}+\delta_{p\alpha+\beta,0}(p\alpha_1K_1+p\alpha_2K_2)]=$$
$$\sum_{k\in\mathbb{Q}^*}\sigma_{k\beta}[\det(k\beta,\alpha)t^{\alpha+k\beta}+\delta_{\alpha+k\beta,0}(\alpha_1K_1+\alpha_2K_2)]. \tag{16}$$

Take $\beta\notin\mathbb{Z}(\alpha)$ in (16), then

$$\sum_{p\in\mathbb{Q}^*}\lambda_{p\alpha}\det(\beta,p\alpha)t^{p\alpha+\beta}=\sum_{k\in\mathbb{Q}^*}\sigma_{k\beta}\det(k\beta,\alpha)t^{\alpha+k\beta},$$

which forces that

$$\lambda_{p\alpha}\det(\beta,p\alpha)=0,\sigma_{p\beta}\det(p\beta,\alpha)=0,\quad p\neq0,\pm1;$$
$$\lambda_\alpha\det(\beta,\alpha)=\sigma_\beta\det(\beta,\alpha).$$

It is easy to see that

$$\begin{cases}\lambda_{p\alpha}=0,\sigma_{p\beta}=0, & p\neq0,\pm1;\\ \lambda_\alpha=\sigma_\beta.\end{cases} \tag{17}$$

Using (17), we get $\lambda_\alpha=\sigma_\varepsilon$ for $\alpha\neq l\varepsilon$, and $\lambda_{l\varepsilon}=\sigma_\eta=\sigma_\varepsilon$, where $l\in\mathbb{Z}^*$. So

$$\lambda_\alpha=\sigma_\varepsilon,\quad\forall\alpha\in G.$$

Similarly, $\sigma_\beta=\lambda_\varepsilon=\sigma_\varepsilon$ for $\beta\neq l\varepsilon$, and $\sigma_{l\varepsilon}=\lambda_\eta=\sigma_\varepsilon$, where $l\in\mathbb{Z}^*$. So

$$\sigma_\beta=\sigma_\varepsilon,\quad\forall\beta\in G.$$

Therefore,

$$\lambda_\alpha=\sigma_\alpha=\sigma_\varepsilon,\quad\forall\alpha\in G.$$

Set $\sigma_\varepsilon=a$. Then (13) and (8) become

$$f(t^\alpha)=at^\alpha+\sum_{i=1}^4x_i^\alpha K_i, \tag{18}$$

$$f(E_\alpha)=\sum_{s\in\mathbb{Q}^*}\rho_{s\alpha}t^{s\alpha}+aE_\alpha+\sum_{i=1}^4y_i^\alpha K_i \tag{19}$$

via (15) and (17).

By (19), we can obtion

$$[f(E_\alpha),E_\beta]=\sum_{s\in\mathbb{Q}^*}[\rho_{s\alpha}\det(\beta,s\alpha)t^{s\alpha+\beta}+\delta_{s\alpha+\beta,0}(s\alpha_1K_1+s\alpha_2K_2)]+a[E_\alpha,E_\beta],$$
$$[E_\alpha,f(E_\beta)]=\sum_{s\in\mathbb{Q}^*}\rho_{s\beta}[\det(s\beta,\alpha)t^{s\beta+\alpha}+\delta_{s\beta+\alpha,0}(\alpha_1K_1+\alpha_2K_2)]+a[E_\alpha,E_\beta].$$

Since $[f(E_\alpha),E_\beta]=[E_\alpha,f(E_\beta)]$, we get

$$\sum_{s\in\mathbb{Q}^*}[\rho_{s\alpha}\det(\beta,s\alpha)t^{s\alpha+\beta}+\delta_{s\alpha+\beta,0}(s\alpha_1K_1+s\alpha_2K_2)]=$$
$$\sum_{s\in\mathbb{Q}^*}\rho_{s\beta}[\det(s\beta,\alpha)t^{s\beta+\alpha}+\delta_{s\beta+\alpha,0}(\alpha_1K_1+\alpha_2K_2)]. \tag{20}$$

Take $\beta\notin\mathbb{Z}(\alpha)$ in (20), then

$$\sum_{s\in\mathbb{Q}^*}\rho_{s\alpha}\det(\beta,s\alpha)t^{s\alpha+\beta}=\sum_{s\in\mathbb{Q}^*}\rho_{s\beta}\det(s\beta,\alpha)t^{\alpha+s\beta},$$

which forces that

$$\rho_{s\alpha}\det(\beta,s\alpha)=0,\quad s\neq0,\pm1;$$

$$\rho_\alpha \det(\beta,\alpha)=\rho_\beta \det(\beta,\alpha), \quad s=1.$$

It is easy to see that

$$\begin{cases}\rho_{s\alpha}=0, & s\neq 0,\pm 1;\\ \rho_\alpha=\rho_\beta.\end{cases} \tag{21}$$

Then $\rho_\alpha=\rho_\varepsilon$ for $\alpha\neq l\varepsilon$, and $\rho_{l\varepsilon}=\rho_\eta=\rho_\varepsilon$, where $l\in\mathbb{Z}^*$. So

$$\rho_\alpha=\rho_\varepsilon, \quad \forall\alpha\in G. \tag{22}$$

Set $b=\rho_\varepsilon$. According to (21) and (22),

$$f(E_\alpha)=bt^\alpha+aE_\alpha+\sum_{i=1}^{4} y_i^\alpha K_i, \quad \forall\alpha\in G. \tag{23}$$

Therefore, the theorem holds via (18) and (23).

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一类 Heisenberg - Virasoro 李代数上的交换线性映射

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摘　要：根据交换线性映射的性质，计算秩为 2 的 Heisenberg - Virasoro 李代数上的交换线性映射，以确定此李代数的形心.

关键词：秩为 2 的 Heisenberg - Virasoro 李代数；交换线性映射；中心

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